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**B-T-SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce **b-t-sets, **b-t*-sets, **b-B-sets, **b-B*-sets, **b-semi open sets, **b-pre open sets and discuss some properties of the above sets.

KEYWORDS: **b-open sets, B-sets, B*-sets, t-sets, t*-sets

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INTRODUCTION

Levine [**9**] introduced the notion of semi open sets and semi-continuity in topological spaces. Andrijevic [**3**] introduced a class of generalized open sets in topological spaces. Mashhour [**10**] introduced pre open sets in topological spaces. The class of b-open sets is contained in the class of semi open and pre open sets. Tong [**13**] introduced the concept of t-set and B-set in topological spaces. The class of *b-open sets is both semi-open and pre open. The class of **b-open sets is both α -open and β -open. Indira, Rekha [**6**] introduced the concept of *b-open set, **b-open set, t*-set and B*-set in topological spaces. In this paper we introduce the notion of **b-t-set, **b-t*-set, **b-B-set, **b-B*-set, **b-semi open and **b-pre open. All throughout this paper (X, τ) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of A and the interior of A will be denoted by Cl(A) and Int(A), respectively.

PRELIMINARIES

Definition 2.1: A subset A of a space X is said to be:

- Semi open [**9**] if $A \subseteq Cl(Int(A))$
- Pre open [**10**] if $A \subseteq Int(Cl(A))$
- b-open [**3**] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$
- *b-open [**6**] if $A \subseteq Cl(Int(A)) \cap Int(Cl(A))$
- b**-open [**4**] if $A \subseteq Int(Cl(Int(A))) \cup Cl(Int(Cl(A)))$
- **b-open [**6**] if $A \subseteq Int(Cl(Int(A))) \cap Cl(Int(Cl(A)))$

Lemma 2.2. [1,4,6] If A is a subset of a space (X, τ), then

- $sInt(A) = A \cap Cl(Int(A))$
- $pInt(A) = A \cap Int(Cl(A))$
- $b^{**}Int(A) = \alpha\,Int(A) \cup \beta\,Int(A)$
- $^{**}bInt(A) = \alpha\,Int(A) \cap \beta\,Int(A)$

Definition 2.3: A subset A of a space X is called:

- t-set [**13**] if Int(A) = Int(Cl(A)).
- B-set [**13**] if A = U ∩ V, where U $\in$ τ and V is a t-set.
- t*-set [**6**] if Cl(A) = Cl(Int(A)).
- B*-set [**6**] if A = U ∩ V, where U $\in$ τ and V is a t*-set.

Definition 2.4: A subset A of a space X is called [**1**]:

- b-t-set if Int(A)=Int(bCl(A))
- b-t*-set if Cl(A) = Cl(bInt(A)).
- b-B-set if A=U∩V, where U $\in$ τ and V is a b-t-set
- b-B*-set if A=U∩V, where U $\in$ τ and V is a b-t*-set
- b-semi open if A $\subseteq$ Cl(bInt(A))
- b-pre open if A $\subseteq$ Int(bCl(A))

GENERALIZED **B-CLOSED SETS

Definition 3.1: Let X be a space. A subset A of X is called a generalized **b-closed set if **bCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

Example 3.2: Let X={a,b,c,d} and τ ={X, ϕ ,{c},{d},{c,d},{a,c,d},{b,c,d}}

The collection of generalized **b-closed sets ={X, ϕ ,{a},{b},{a,b},{a,b,c},{a,b,d}}

Definition 3.3: The complement of a generalized **b-closed set is called generalized **b-open set.

From the example 3.2, The collection of generalized **b-open sets={X, ϕ ,{b,c,d},{a,c,d},{c,d},{d},{c}}

Theorem 3.4: For a subset A of a space(X, τ), the following are equivalent:

- A is **b-closed
- A is g-**b-closed and locally **b-closed.

Proof

To prove: (1) $\Rightarrow$ (2)

Let A be **b-closed. Then A=A∩X where A is **b-closed and X is open. Therefore A is locally **b-closed.

To Prove: A is generalized **b- closed.

Let A $\subseteq$ U and U is open. Then **bCl(A) $\subseteq$ U [since A is **b-closed]. Therefore A is generalized **b- closed.

To prove: (2) $\Rightarrow$ (1)

Let A be locally **b-closed and generalized **b-closed.

To Prove: A is **b-closed.

Since A is locally **b-closed. Therefore, there exist an open set U such that A =U∩**bCl(A) (1)

$\Rightarrow$ A $\subseteq$ U and A $\subseteq$ **bCl(A) (2)

$\Rightarrow$ **bCl(A) $\subseteq$ U

$\Rightarrow$ **bCl(A) $\subseteq$ U∩**bCl(A) =A [From(1)]. Therefore **bCl(A) $\subseteq$ A . Hence A is **b-closed.

**B-T-SETS

Definition 4.1: A subset A of a space X is called:

- **b-t-set if Int(A)=Int(**bCl(A))
- **b-t*-set if Cl(A)=Cl(**bInt(A))
- **b-B-set if A=U∩V, where U $\in$ τ and V is a **b-t-set
- **b-B*-set if A=U∩V, where U $\in$ τ and V is a **b-t*-set
- **b-semi open if A $\subseteq$ Cl(**bInt(A))
- **b-pre open if A $\subseteq$ Int(**bCl(A))

Example 4.2: Let X={a,b,c,d} and τ ={X, ϕ ,{c},{d},{c,d},{a,c,d},{b,c,d}}

The collection of **b-open sets ={X, ϕ ,{c},{d},{c,d},{a,c,d},{b,c,d}}

The collection of **b-closed sets ={X, ϕ ,{a,b,d},{a,b,c},{a,b},{b},{a}}

The collection of **b-t-sets ={X, ϕ ,{a},{b},{c},{d},{a,b},{a,c},{b,c},{b,d},{a,b,c},{a,b,d}}

The collection of **b-t*-sets ={X, ϕ ,{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,c,d},{a,b,d},{b,c,d}}

The collection of **b-B-sets ={X, ϕ ,{a},{b},{c},{d},{a,b},

{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,c,d},{a,b,d},{b,c,d}}

The collection of **b-B*-sets ={X, ϕ ,{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,c,d},{a,b,d},{b,c,d}}

The collection of **b-semi open sets ={X, ϕ ,{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,c,d},{a,b,d},{b,c,d}}

The collection of **b-pre open sets ={X, ϕ ,{c},{d},{c,d},{a,c,d},{b,c,d}}

Results 4.3

- Every **b-closed set is a **b-t-set.
- Every **b-open set is a **b-t*-set.
- Every open set is a **b-pre open set.

Example 4.4: From the example 4.2,

- {d} is a **b-t-set, but not a **b-closed set.
- {a,c} is a **b-t*-set, but not a **b-open set.

Definition 4.5: The complement of a **b-semi open (**b-pre open) set is called a **b-semi closed (**b-pre closed) set.

From the example 4.2,

The collection of **b-semi closed sets ={X, ϕ ,{a,b,d},{a,b,c},{b,d},{b,c},{a,d},{a,c},{a,b},{b},{c},{a}}

The collection of **b-pre closed sets ={X, ϕ ,{a,b,d},{a,b,c},{a,b},{b},{a}}

Note 4.6

S(**bCl(A)) is the intersection of all **b-semi closed sets containing A.

S(**bInt(A)) is the union of all **b-semi open sets contained in A.

Theorem 4.7: For subsets A and B of a space (X, τ), the following properties hold:

- A is a **b-t*-set iff it is **b-semi open.
- If A and B are **b-t*-sets, then A ∪ B is a **b-t*-set.

Proof

- Let A be a **b-t*-set.

  Then Cl(A)=Cl(**bInt(A)). Therefore A ⊆ Cl(**bInt(A)). Hence A is **b-semi open.

  Conversly, If A is **b-semi open.

  Then A ⊆ Cl(**bInt(A)), Cl(A) ⊆ Cl(**bInt(A)) (1)

  Since Cl(**bInt(A)) ⊆ Cl(A) (2)

  Hence A is a **b-t*-set.

- Let A and B be **b-t*-sets.

  Since **bInt(A ∪ B) ⊆ (A ∪ B)

  Cl(**bInt(A ∪ B)) ⊆ Cl(A ∪ B) (1)

  Cl(**bInt(A ∪ B)) ⊃ Cl(**bInt(A) ∪ **bInt(B)) = Cl(A) ∪ Cl(B) = Cl(A ∪ B) (2)

  From (1) & (2), Cl(**bInt(A ∪ B)) = Cl(A ∪ B). Hence A ∪ B is a **b-t*-set.

Theorem 4.8: For subsets A and B of a space (X, τ), the following properties hold:

- A is a **b-t-set if and only if it is **b-semi closed.
- If A is **b-closed, then it is a **b-t-set.
- If A and B are **b-t-sets, then A∩B is a **b-t-set.

Proof

- Let A be a **b-t-set. Therefore Int(**bCl(A)) ⊆ Int(A)

  Int(**bCl(A)) ⊆ A.Hence A is **b-semi closed.

Conversly, If A is **b-semi closed.

Then Int(**bCl(A)) $\subseteq$ A $\Rightarrow$ Int(Int(**bCl(A))) $\subseteq$ Int(A) $\Rightarrow$ Int(**bCl(A)) $\subseteq$ Int(A) (1)

Since Int(A) $\subseteq$ Int(**bCl(A)) (2)

From (1) & (2), Int(A)=Int(**bCl(A)) $\Rightarrow$ A is a **b-t-set.

- Let A be a **b-closed set. Then A=**bCl(A) $\Rightarrow$ Int(A)=Int(**bCl(A)) $\Rightarrow$ A is a **b-t-set.
- Let A and B be **b-t-sets. Since A $\cap$ B $\subseteq$ **bCl(A $\cap$ B)

  $\Rightarrow$ Int(A $\cap$ B) $\subseteq$ Int(**bCl(A $\cap$ B)) (1)

  Int(**bCl(A $\cap$ B)) $\subseteq$ Int(**bCl(A) $\cap$ **bCl(B)) = Int(A) $\cap$ Int(B) = Int(A $\cap$ B) (2)

  From (1) & (2), Int(A $\cap$ B)= Int(**bCl(A $\cap$ B)) $\Rightarrow$ A $\cap$ B is a **b-t-set.

Theorem 4.9: For a subset A of a space (X, τ), the following properties hold:

- If A is t-set then it is **b-t-set.
- If A is **b-t-set then it is **b-B-set.
- If A is B-set then it is **b-B-set.

Proof

- Let A be a t-set. Then Int(A)=Int(Cl(A)) $\Rightarrow$ A is closed.

  Since every closed set is **b-closed. $\Rightarrow$ Int(A)=Int(**bCl(A)) . Hence A is a **b-t-set.
- Let A be a **b-t-set. Let U $\in$ τ be an open set containing A & V=A be a **b-t-set

  containing A. $\Rightarrow$ A=U∩V $\Rightarrow$ A is a **b-B-set.
- Let A be a B-set. Then A=U∩V where U $\in$ τ and V is a t-set. Since every t-set is a **b-t-set.

  $\Rightarrow$ A is a **b-B-set.

Theorem 4.10: For a subset A of a space (X, τ), the following properties hold:

- If A is a t*-set then it is **b-t*-set.
- If A is a **b-t*-set then it is **b-B*-set.
- If A is a B*-set then it is **b-B*-set.

Proof

- Let A be a t*-set. Then Cl(A)=Cl(Int(A)). $\Rightarrow$ A is open.

  Since every open set is **b-open. $\Rightarrow$ Cl(A)=Cl(**bInt(A)) $\Rightarrow$ A is a **b-t*-set.
- Let A be a **b-t*-set. Let U $\in$ τ be an open set containing A&V=A be a **b-t*-set

  containing A. $\Rightarrow$ A=U∩V $\Rightarrow$ A is a **b-B*-set.
- Let A be a B*-set. Then A=U∩V where U $\in$ τ and V is a t*-set

Since every t*-set is a **b-t*-set. Hence A is a **b-B*-set.

Theorem 4.11: For a subset A of a space (X, τ), the following are equivalent:

- A is open.
- A is **b-pre open and a **b-B-set.

Proof

To Prove: (1) $\Rightarrow$ (2)

Let A be open. $\Rightarrow$ A =Int(A) .Since A $\subseteq$ **bCl(A) $\Rightarrow$ A $\subseteq$ Int(**bCl(A) $\Rightarrow$ A is **b-pre open.

Let U=A $\in$ τ and V=X be a **b-t-set containing A. $\Rightarrow$ A=U∩V $\Rightarrow$ A is a **b-B-set.

To Prove: (2) $\Rightarrow$ (1)

Let A be **b-pre open and a **b-B-set. Since A is **b-B-set.

We have A=U∩V, where U is an open set and Int(V)=Int(**bCl(V))

Since A is also **b-pre open.

$\Rightarrow$ A $\subseteq$ Int(**bCl(A)) = Int(**bCl(U∩V))

$\subset$ Int(**bCl(U)∩**bCl(V)) = Int(**bCl(U))∩Int(**bCl(V))

= Int(**bCl(U))∩Int(V)

Therefore A =U∩V $\subset$ Int(**bCl(U))∩Int(V)

Consider U∩V = (U∩V)∩U $\subset$ [Int(**bCl(U))∩Int(V)]∩U = [Int(**bCl(U))∩U]∩Int(V) = U∩Int(V)

$\Rightarrow$ U∩V $\subset$ U∩Int(V).Therefore A=U∩V= U∩Int(V) $\Rightarrow$ A is open.

Lemma 4.12: Let A be an open subset of a space X.

Then $**bCl(A) = Cl(A)$ and $Int(**bCl(A)) = Int(Cl(A))$.

Proof

Let A be an open set. Then $A = Int(A)$

$$**bCl(A) = \alpha Cl(A) \cup \beta Cl(A)$$

$$= [A \cup Cl(Int(Cl(A)))] \cup [A \cup Int(Cl(Int(A)))] = A \cup [Cl(Int(Cl(A))) \cup Int(Cl(Int(A)))]$$

$$= A \cup [Int(Cl(A)) \cup Cl(Int(Cl(A)))]$$

$$= A \cup Cl(Int(Cl(A))) \subset A \cup Cl(Cl(Cl(A)))[\because Int(Cl(A)) \subset Cl(Cl(A))]$$

$$\subset A \cup Cl(A) = Cl(A) \Rightarrow **bCl(A) \subset Cl(A)$$

$$\because Cl(A) \subset **bCl(A) \Rightarrow **bCl(A) = Cl(A)$$

$$\Rightarrow Int(**bCl(A)) = Int(Cl(A))$$

Proposition 4.13: For a subset A of a space (X, τ), the following are equivalent:

- A is regular open.
- $A = Int(**bCl(A))$
- A is **b-pre open and a **b-t-set.

Proof

To prove: (1) $\Rightarrow$(2)

Let A be regular open. $\Rightarrow A = Int(Cl(A))$ (1)

By lemma 4.12 , $**bCl(A) = Cl(A) \Rightarrow Int(**bCl(A)) = Int(Cl(A))$

$\Rightarrow Int(**bCl(A)) = A$ [From (1)] $\Rightarrow A = Int(**bCl(A))$

To prove: (2) $\Rightarrow$(3)

Given $A = Int(**bCl(A))$ (2)

$\Rightarrow A \subseteq Int(**bCl(A))$ & $Int(**bCl(A)) \subseteq A \Rightarrow A$ is **b-pre open.

From(2),

$Int(A) = Int(Int(**bCl(A))) \Rightarrow Int(A) = Int(**bCl(A))$

$\Rightarrow A$ is a **b-t-set.

To prove: (3) $\Rightarrow$(1)

Let A be **b-pre open and a **b-t-set. Since A is **b-pre open.

$\Rightarrow A \subseteq Int(**bCl(A))$ (3)

Since A is a **b-t-set. $\Rightarrow Int(A) = Int(**bCl(A))$ (4)

From (3) & (4), $A \subseteq Int(**bCl(A)) = Int(A) \subseteq A \Rightarrow A \subseteq Int(**bCl(A)) \subseteq A$

$\Rightarrow A = Int(**bCl(A))$

$\Rightarrow A = Int(Cl(A))$ [Since A is open]

$\Rightarrow A$ is regular open.

Definition 4.14: A subset A of a topological space X is called s-**b-generalized closed if $s(**bCl(A)) \subseteq U$

Whenever $A \subseteq U$ and U is **b-pre open.

- $s(**bCl(A))$ is the intersection of all **b-semi closed sets containing A.

Example 4.15

Let $X=\{a,b,c,d\}$

$\tau=\{X,\phi,\{c\},\{d\},\{c,d\},\{a,c,d\},\{\{b,c,d\}\}$

The collection of s-**b-generalized closed sets =

$$\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\}\}$$

Theorem 4.16: For a subset A of a topological space X , the following are equivalent:

- A is regular open.
- A is **b-pre open and s-**b-generalized.

Proof

To prove: (1)$\Rightarrow$(2)

Let A be regular open.

$\Rightarrow A$ is **b-pre open [By Proposition 4.13]

$\Rightarrow A\subseteq Int(**bCl(A))$

$s(**bCl(A))=A\cup Int(**bCl(A))$

$=Int(**bCl(A))$

$=Int(Cl(A))=A$ [$\because A$ is regular open]

$\Rightarrow s(**bCl(A))=A$

$\Rightarrow A$ is s-**b-generalized closed.

To prove: (2)$\Rightarrow$(1)

Let A be **b-pre open and s-**b-generalized closed.

To prove: A is regular open.

Since A is **b-pre open.

Let $U=A$

$\Rightarrow s(**bCl(A))\subseteq A$ (1)

Since $A\subseteq s(**bCl(A))$ (2)

From (1) & (2)

$A=s(**bCl(A))$

$\Rightarrow A$ is **b-semi closed.

Therefore $Int(**bCl(A)) \subseteq A$ (3)

Since A is **b-pre open.

$\Rightarrow A \subseteq Int(**bCl(A))$ (4)

From (3) & (4)

$A = Int(**bCl(A))$

$\Rightarrow A$ is regular open.

Theorem 4.17: A set A is a **b-t-set iff its complement is a **b-t*-set.

Proof

Let A be a **b-t-set.

Then Int(A)=Int(**bCl(A))

$\Leftrightarrow$ X-Int(A)=X-Int(**bCl(A))

$\Leftrightarrow$ Cl(X-A)=Cl(X-**bCl(A))

$\Leftrightarrow$ Cl(X-A)=Cl(**bInt(X-A))

$\Leftrightarrow$ Cl(A^c)=Cl(**bInt(A^c))

$\Leftrightarrow$ A^c is a **b-t*-set.

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